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SUMMARY

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The minimum weight analysis of moderate length, grid stiffened cylinders under axial compression is presented based on the use of orthotropic cylinder theory. The results are obtained for both rectangular and tee cross section stiffeners.

The minimum weight results obtained herein are compared with isotropic, ring stiffened, sandwich and pressure stabilized cylinders to establish the comparative efficiencies of each type over a broad range of the governing structural loading parameter. Finally, design data on current and projected launch vehicles indicate that all such designs fall within a very narrow range of the structural loading parameter. This observation permits a set of generalized conclusions to be drawn concerning the solution of efficient stiffening systems and materials for launch vehicle designs.

AUTHOR

LIST OF SYMBOLS

b_f	=	flange width, in
b_s	=	longitudinal stiffener spacing, in
b_w	=	web height, in
B	=	extensional rigidity, lb/in
C	=	isotropic cylinder buckling coefficient
d	=	cylinder diameter, in
D	=	flexural rigidity, in-lb
E	=	modulus of elasticity, psi
L_t	=	ring spacing, in
n	=	ratio of web to flange weight
N	=	loading, lb/in
R	=	cylinder radius, in
S_i	=	weight of stiffener relative to skin
t	=	isotropic cylinder thickness, in
t_f	=	flange thickness, in
t_i	=	effective thickness, in
t_s	=	skin thickness, in
t_w	=	web thickness, in
α	=	mode index
σ	=	stress, psi
$\bar{\sigma}$	=	effective stress, psi
σ_a	=	applied stress, psi
σ_{co}	=	wide panel critical stress, psi
σ_g	=	general instability critical stress, psi

σ_s	=	sheet critical stress, psi
Σ	=	solidity
ν	=	poisson's ratio

SUBSCRIPTS

1	=	longitudinal direction
2	=	circumferential direction
o	=	optimum value

$i = 1, 2$

MINIMUM WEIGHT DESIGN OF STIFFENED CYLINDERS FOR LAUNCH VEHICLE APPLICATIONS

1. INTRODUCTION

The minimum weight design of stiffened cylinders under axial compression is of fundamental importance in the design of launch vehicles. As a consequence of recent developments concerning the stability of orthotropically stiffened cylinders under various loading conditions, sufficient theory and experimental data exist to permit a first approach to the minimum weight design of grid stiffened cylinders of a somewhat restrictive nature.

In this report, the analyses of grid stiffened systems of the circumferential rather than the longitudinal type are presented. Although the grid stiffening system is composed of circumferential and longitudinal elements, the denotation "circumferential type" indicates that the stability behavior corresponds to a stiffening system that is predominately circumferential in nature.

Part 2 contains the analysis of stiffening elements that are rectangular in cross section. In Part 3, the analysis is conducted for the more efficient Tee shaped longitudinal stiffener. The minimum weight design results of Parts 2 and 3 are compared in Part 4 with other axially compressed cylinder designs such as isotropic, sandwich, pressure stabilized and ring stiffened. In addition, design data on current and projected launch vehicles are used to draw pertinent design conclusions on the relative efficiencies of various types of stiffening systems and materials.

2. RECTANGULAR CROSS SECTION STIFFENERS

Stability Considerations

Cylinders of interest in aerospace applications are of moderate length, neither so long that they buckle as Euler columns nor so short that they buckle as width panels. Such moderate length cylinders can buckle in two modes. Both unstiffened and longitudinally stiffened cylinders display the characteristic diamond shape buckling patterns of the asymmetric mode while ring stiffened and certain grid stiffened cylinders generally will display the axisymmetric sinusoidal buckle patterns as shown in Reference 1.

It is a generally accepted axiom in optimum design that all possible forms of buckling in the structure occur simultaneously (2, 3). For the grid stiffened cylinder with rectangular cross-section stiffeners shown in Figure 1 this involves general instability of the cylinder in the axisymmetric mode, buckling of the structure between rings as a wide column, buckling of the sheet between the stiffeners and buckling of the longitudinal stiffeners. The ring stiffeners theoretically carry no load and serve only to constrain the cylinder to buckle axisymmetrically.

It was indicated in Reference 1 that the stiffening system behavior can be characterized as the circumferential or longitudinal type depending upon the relative flexural and extensional rigidities of the two stiffening systems. A mode index which characterizes this behavior is in the form:

$$\alpha = \frac{B_1 D_2}{D_1 B_2} \tag{1}$$

Axisymmetric buckling will occur when $\alpha > 1$ which characterizes circumferential stiffening while asymmetic buckling governs for $\alpha < 1$ and characterizes longitudinal stiffening systems. The actual value of α to be used for a particular circumferential stiffening system cannot be specified precisely at this time although available test data indicate that α should have a value greater than unity. For the circumferential type stiffened cylinder considered here, the mode index is given by

$$\alpha = (b_{w2}/b_{w1})^2 (S_2/S_1) ([1 + S_1]/[1 + S_2])^2 ([4 + S_2]/[4 + S_1]) \tag{2}$$

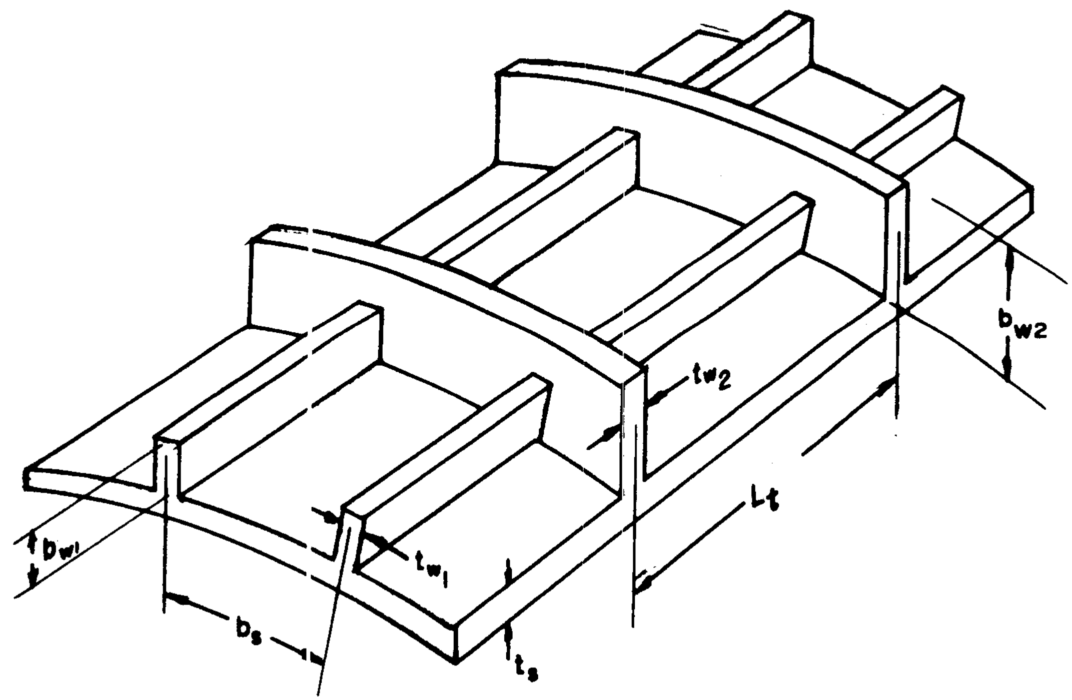


FIG. I

GRID STIFFENED CYLINDER CONFIGURATION

Equation (2) indicates that relatively small changes in the ratio of the two stiffener heights will have a much more significant effect on α than comparable changes in the ratio of the two stiffening system weights. On this basis and because the mathematical procedure for optimization would be greatly simplified, the assumption was made that the stiffening systems have the same weight but could vary in both the height and the thickness of the stiffeners.

Since the design basis for the cylinder is stability and since there have been many theoretical approaches to cylinder stability under axial compression, it should be pointed out that the particular theoretical formulation used here for general instability has had experimental confirmation. The linear theory developed by Gerard (1, 4) and Becker and Gerard (5) has had high predictive value for buckling stresses of ring stiffened cylinders. Recent experiments (6) show that test cylinders designed to buckle axisymmetrically did so at stresses which were on the average 94 percent of those predicted by theory; this is a rather remarkable correlation in an area where in the past theoretical stress predictions have shown wide divergence from the test results. Although tests were not conducted with grid stiffened cylinders, the excellent theoretical correlation for the ring stiffened cylinders leads one to have some confidence in applying the theory to cylinders with the more complicated stiffener configuration.

Minimum Weight Design Analysis

The quantitative criteria that the cylinder be of moderate length are given in Reference (7) and it is assumed that the cylinder will be in this range. The material of construction, the loading and the cylinder diameter will be considered fixed parameters. It will also be assumed that the maximum design stress is elastic, i.e., elastic buckling relations will hold.

Since the basis for the design is that all possible forms of buckling occur simultaneously, the various independent critical stresses are combined with the applied stress to give a stress relation which contains only the geometric term S_1 in addition to the fixed parameters. In the second step of the process the solidity relation is set up using the combined stress equation and this is minimized with respect to S_1. The solidity is a non-dimensional representation of the structural weight, while the geometric parameter S_1 represents the weight of the longitudinal stiffening system relative to the skin.

The critical stress for general elastic instability of a grid stiffened cylinder with equal weight rectangular cross-section longitudinal and ring stiffeners can be given in the form (allowing $S_1 = S_2 = S$)

$$\sigma_g = \frac{1.404\pi^2 E}{12(1-\nu^2)^{\frac{1}{2}}} \; \frac{S^{\frac{1}{2}}(4+S)^{\frac{1}{2}}}{(1+S)} \; (b_{w1}/d) \tag{3}$$

The sheet between the stiffeners is assumed to be simply supported along all four edges and to buckle at a stress given by:

$$\sigma_s = \frac{4\pi^2 E}{12(1-\nu^2)} \; (t_s/b_s)^2 \tag{4}$$

The applied stress

$$\sigma_a = \frac{N}{t_s(1+S)} \tag{5}$$

Using the dimensional analysis procedure described in Reference (8), Equations (3), (4) and (5) can be combined in the following fashion to give the combined stress relation

$$\bar{\sigma} = \sigma_g^{2/5} \, \sigma_s^{1/5} \, \sigma_a^{2/5} \tag{6}$$

It should be noted that in forming Equation (6), neither the buckling relation for the structure between the rings nor for the longitudinal stiffeners is taken directly into account. The former will be employed subsequently to obtain the ring spacing. For the latter there is a relation between the sheet buckling stress and the stiffener buckling stress in that the stiffener can be considered to be a sheet which is free on one unloaded edge and simply supported on the other three edges. Equating the buckling stresses for sheet and stiffener and rearranging terms

$$\frac{b_{w1}}{b_s} = 3.05 \; \frac{t_{w1}}{t_s} \tag{7}$$

By definition the stiffener weight is given by

$$S = (b_{w1}/b_s)\ (t_{w1}/t_s) \tag{8}$$

Combining Equations (7) and (8) the relation between the stiffener height and the stiffener weight becomes

$$(b_{w1}/b_s)^2 = \frac{S}{3.05} \tag{9}$$

Substituting the appropriate critical stress relations in Equation (6) and using Equation (9) to eliminate the stiffener height factor results in

$$\bar{\sigma} = 0.272\pi\ (\pi/1-\nu^2)^{1/5}\ \frac{S^{2/5}\ (4+S)^{1/5}}{(1 + S)^{4/5}}\ E(N/Ed)^{2/5} \tag{10}$$

The solidity of a stiffened structural cylinder is the ratio of its weight to the weight of a solid cylinder of the same radius. One form of the solidity expression for a grid stiffened cylinder with equal relative weight longitudinals and rings can be given as

$$\Sigma = \frac{t_s}{d}\ (1 + 2S) \tag{11}$$

The sheet thickness in Equation (11) can be eliminated by using Equations (5) and (10) to obtain a solidity expression in terms of S and fixed parameters.

$$\Sigma = 3.58\ \frac{(1 + 2S)}{S^{2/5}(1+S)^{1/5}(4+S)^{1/5}}\ (N/Ed)^{3/5} \tag{12}$$

It is now possible to minimize the solidity with respect to S by $\partial\Sigma/\partial S = 0$. Performing the indicated operation results in

$$S_o = 0.471 \tag{13}$$

Optimum Proportions

With the optimum stiffener weight known, it is now possible to determine the various other optimum parameters.

Solidity: $$\Sigma_o = 6.45\,(N/Ed)^{3/5} \tag{14}$$

Sheet thickness: $$\frac{(t_s)_o}{d} = 0.829\,(N/Ed)^{3/5} \tag{15}$$

Longitudinal Stiffener spacing: $$\frac{(b_s)_o}{d} = 1.55\,(N/Ed)^{2/5} \tag{16}$$

Longitudinal Stiffener height: $$\frac{(b_{w1})_o}{d} = 0.61\,(N/Ed)^{2/5} \tag{17}$$

Longitudinal Stiffener thickness: $$\frac{(t_{w1})_o}{d} = 0.995\,(N/Ed)^{3/5} \tag{18}$$

Ring Spacing: $$\frac{(L_t)_o}{d} = 0.562\,(N/Ed)^{1/5} \tag{19}$$

There are two remaining parameters whose value cannot be determined directly from the analysis thus far: the ring thickness and the ring height. However, the axisymmetric buckling criterion can be invoked to determine the ring height. Using Equation (2) with the assumption that the two stiffening systems have equal weight:

Ring height: $$\frac{(b_{w-2})_o}{d} = 0.61\,\alpha^{\frac{1}{2}}(N/Ed)^{2/5} \tag{20}$$

From the known ring stiffener weight, spacing and height the ring stiffener thickness can finally be found.

Ring thickness: $$\frac{(t_{w-2})_o}{d} = \frac{0.765}{\alpha^{\frac{1}{2}}}\,(N/Ed)^{2/5} \tag{21}$$

The values of the various optimized parameters which are independent of the mode index (Eq. (1)) are shown in Figure 2 as functions of the loading parameter. The ring heights and thicknesses are shown in Figure 3 as a function of the loading parameter for several values of α. The value $\alpha = 1$ represents the lower limit for the ring height and the upper limit for the ring thickness. It is necessary to use values in excess of unity, to assure that axisymmetric buckling will occur, however, it should be noted that sufficient experimental evidence does not yet exist to assign a specific value to α which will hold for all orthotropic cylinders. The lines at 45° in the figure represent yield strength cut off values of the parameter for high strength materials ($E/\sigma_{cy} = 100$).

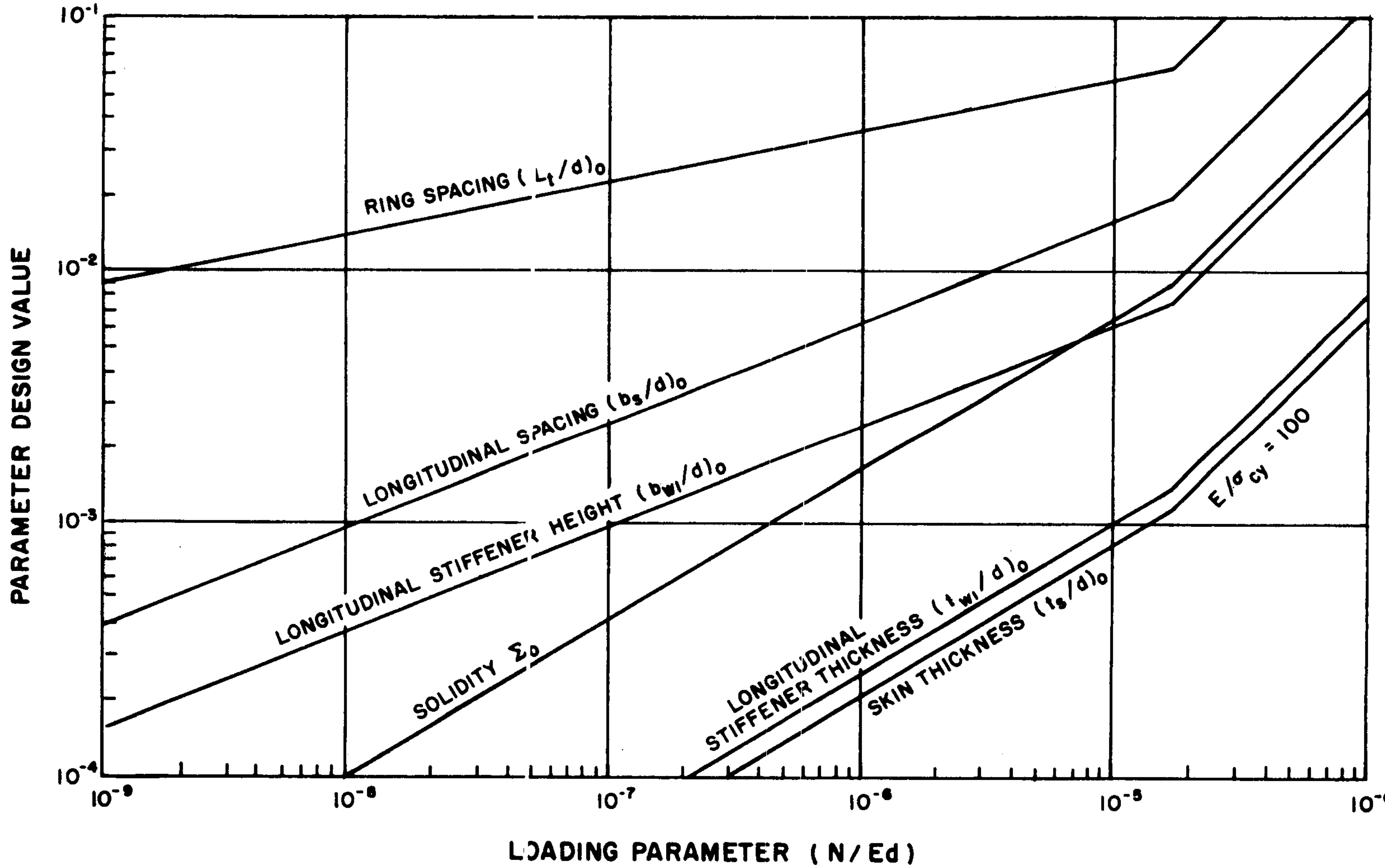


FIG.2

STIFFENER SPACING, LONGITUDINAL STIFFENER GEOMETRY AND SOLIDITY FOR OPTIMUM DESIGN

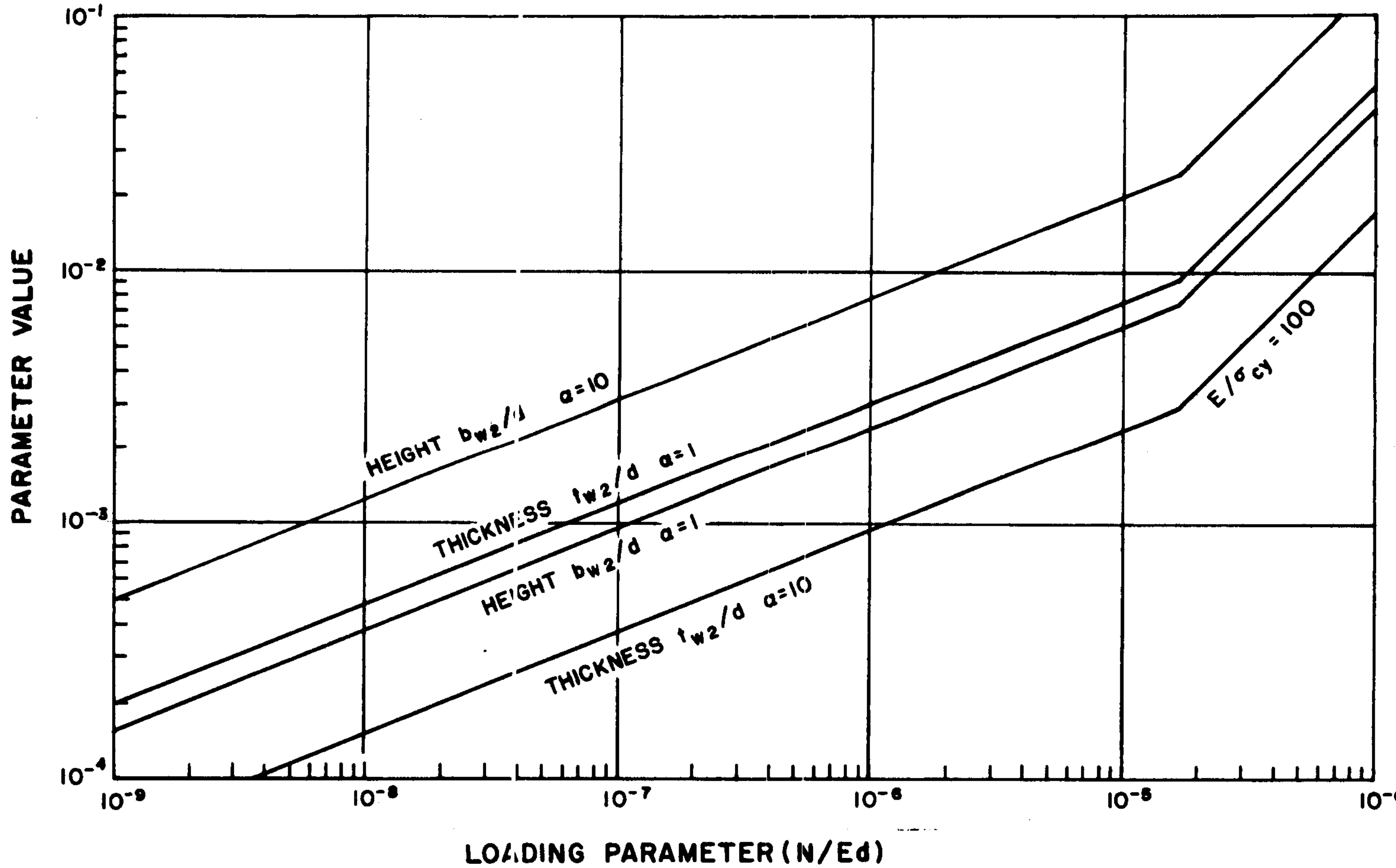


FIG. 3

RING DESIGN GEOMETRY

For a visualization of a typical stiffening system in the true proportions, Figure 4 was prepared using a loading index value $(N/Ed) = 10^{-7}$ which is a lower limit for launch vehicle structures.

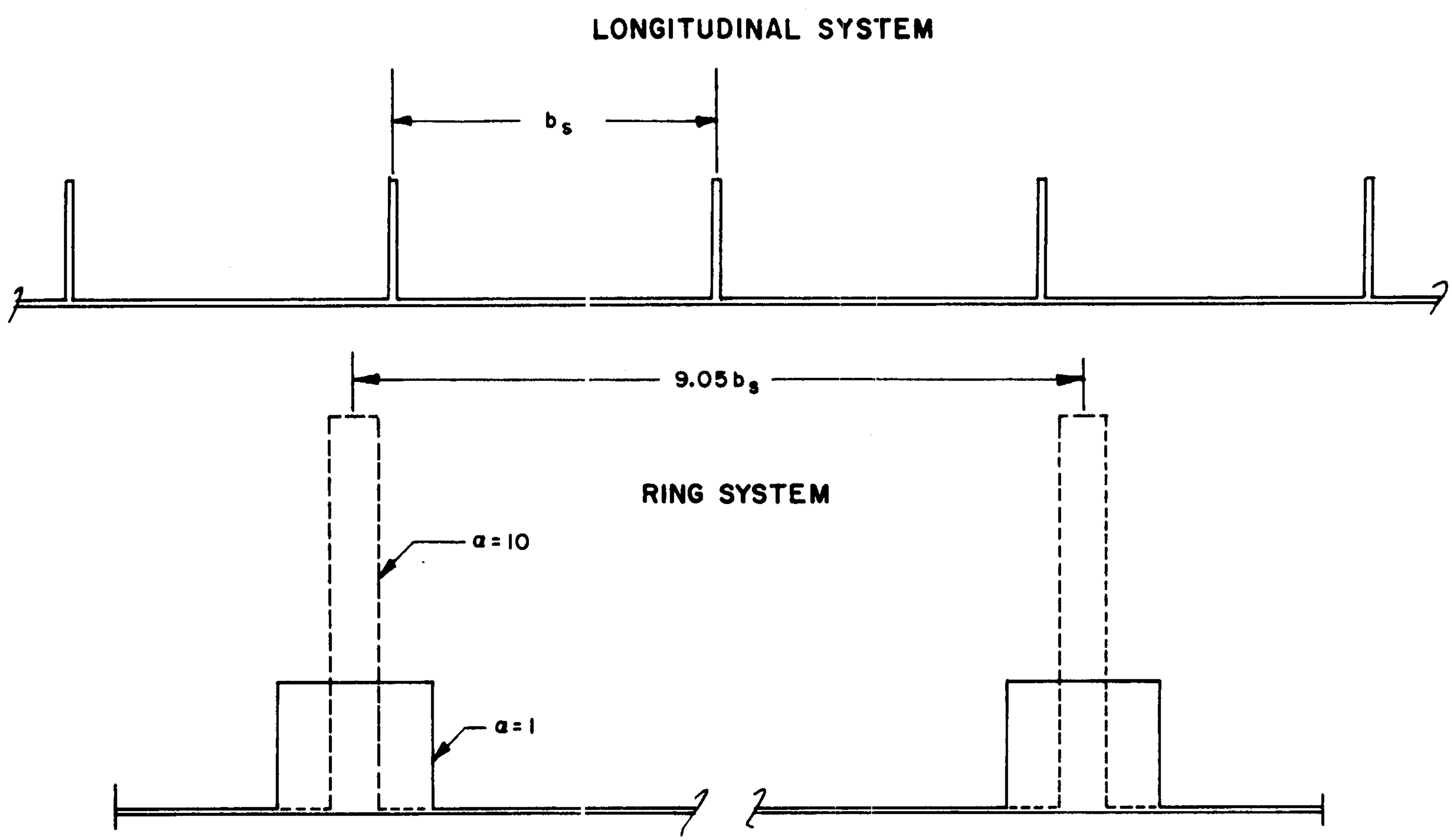


FIG. 4

OPTIMUM STIFFENING SYSTEM PROPORTIONS FOR A LOADING PARAMETER VALUE $\left(\frac{N}{Ed}\right) = 10^{-7}$ (ARBITRARY SCALE)

3. LONGITUDINAL TEE STIFFENERS AND EQUAL WEIGHT RINGS

Stability Considerations

In Part 2, the optimum design for grid stiffened cylinders with equal weight, rectangular cross section longitudinals and rings was presented. This part deals with another equal weight system: longitudinal tee stiffeners and equal weight rectangular or tee rings. The optimum design analysis for this configuration differs from that of the simpler rectangular case in that the optimum distribution of material between web and flange in the longitudinal stiffeners is determined as well as the optimum stiffener weight.

The basic assumption in the optimum design of a compression structure is that all forms of instability occur simultaneously. For a grid stiffened cylinder, three major forms of instability occur: general instability of the cylinder, buckling of the skin between the stiffeners, and buckling of the longitudinal stiffening system. The ring stiffeners themselves in an appropriately designed cylinder do not buckle, but serve to constrain the cylinder to buckle in the axisymmetric mode which is characteristic of the circumferential type of stiffened cylinder behavior.

Because of the fact that the rings do not buckle, it is necessary to make an assumption as to the total weight of the ring stiffeners. The most convenient, from the mathematical point of view, is that the relative weight of the ring stiffeners with respect to the skin is the same as that for the longitudinals. This assumption has resulted in the optimum design for rectangular cross section stiffeners presented in Part 2 and it appears at this time to be the only feasible assumption that can be made.

Effective Design Stress

The general instability of a grid stiffened cylinder under axial compression (1) which buckles elastically in the axisymmetric mode is given by

$$\sigma_g = \frac{1.404\,\pi^2\,(1-\nu^2)^{\frac{1}{2}}\,(B_2 D_1)^{\frac{1}{2}}}{12^{\frac{1}{2}}\quad t_1 d} \tag{22}$$

In order for Equation (22) to be valid the mode index given by Equation (1) must have a value of $\alpha > 1$.

The appropriate extensional and flexural rigidities for a tee longitudinal stiffening system and for rectangular or tee cross section rings of the configuration shown schematically in Figure 5 and for substitution in Equation (22) are given by

$$\text{Tee Rings:} \quad B_2 = t_s\,(1 + S_{w2} + S_{f2})\,\frac{E}{1-\nu^2} \tag{23}$$

$$\text{Rectangular Rings:} \quad B_2 = t_s\,(1 + S_2)\frac{E}{1-\nu^2} \tag{24}$$

and

$$\text{Tee Longitudinals:} \quad D_1 = t_s b_{w1}^2 \left[\frac{S_{w1}\,(S_{w1} + 4) + S_{f1}\,(S_{w1} + 3)}{1 + S_{w1} + S_{f1}}\right]\frac{E}{12(1-\nu^2)} \tag{25}$$

where the weight terms are defined as follows:

$$S_{w1} = \frac{b_{w1}\,t_{w1}}{b_s\,t_s} \qquad S_{f1} = \frac{b_{f1}\,t_{f1}}{b_s\,t_s}$$

$$S_{w2} = \frac{b_{w2}\,t_{w2}}{L_t\,t_s} \qquad S_{f2} = \frac{b_{f2}\,t_{f2}}{L_t\,t_s} \qquad S_2 = \frac{b_{w2}\,t_{w2}}{L_t\,t_s} \tag{26}$$

The assumption that the longitudinals and the rings have the same relative weight results in

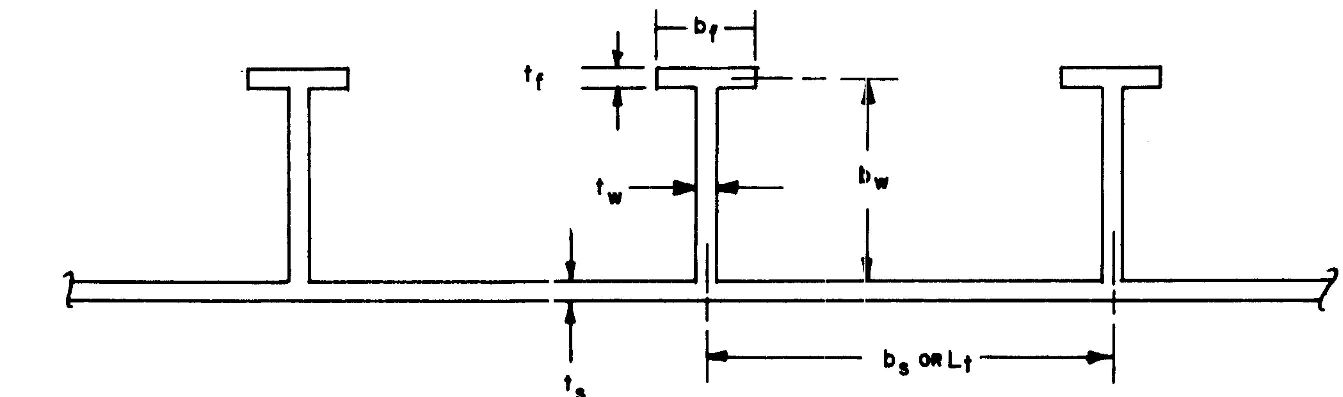
$$S_{w1} + S_{f1} = S_{w2} + S_{f2}$$

or (27)

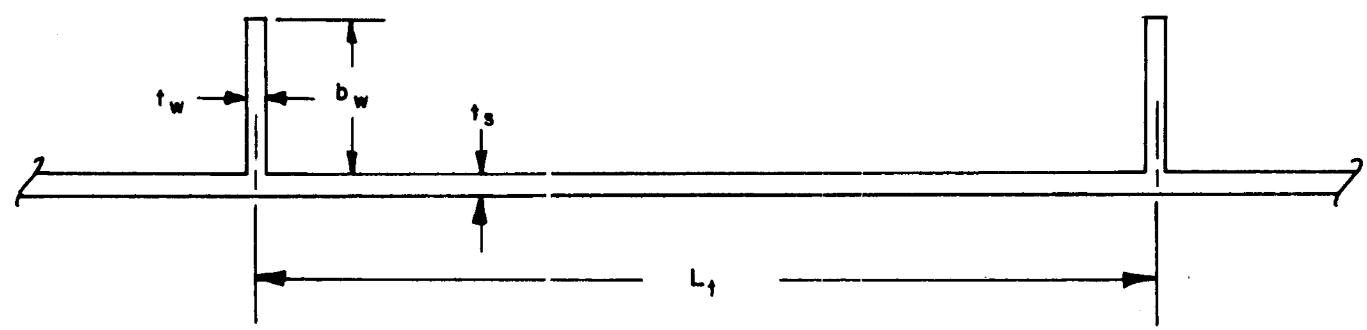
$$S_{w1} + S_{f1} = S_2$$

Substituting Equations (23) to (25) into Equation (22) and using the appropriate form of Equation (27) results in a general instability expression of the form (dropping the subscripts 1 and 2)

$$\sigma_g = \frac{1.404\pi^2 E}{12(1-\nu^2)}\;\frac{S_w(S_w + 4) + 4\,S_f(S_w + 3)}{1 + S_w + S_f}\;\frac{b_w}{d} \tag{28}$$



"TEE" STIFFENER EITHER LONGITUDINAL OR RING



RECTANGULAR RING

FIG.5

STIFFENER CONFIGURATION

It is understood that the weight terms in Equation (28) now refer only to the longitudinal system.

In a grid stiffened cylinder the sheet between the stiffeners can be considered to be simply supported along all four edges. Under axial compression along two opposite edges the buckling stress is given by

$$\sigma_s = \frac{4\pi^2 E}{12(1-\nu^2)} \left(t_s/b_s\right)^2 \tag{29}$$

The buckling of the stiffened structure between the rings as a wide column will be taken into account subsequently to obtain the ring spacing.

The third buckling condition, that of the stiffeners, is not used directly in the formulation of the effective stress equation but rather to make certain substitutions afterward. The forms of the buckling stress relation for the stiffener web, considered to be simply supported at its points of attachment to the skin and that of the half flange considered free on one edge and simply supported at its point of attachment to the web are both similar to Equation (29). Assuming all elements buckle simultaneously at the same stress, the relations given below result:

For the web

$$\left(t_w/b_w\right)^2 = \left(t_s/b_s\right)^2 \tag{30}$$

For the flange

$$0.433 \left(t_f/b_f\right)^2 = \left(t_w/b_w\right)^2 \tag{31}$$

In an optimum design it is assumed that each of the equal separate buckling stresses in turn are equal to the applied loading. The latter is in the form

$$\sigma_a = \frac{N}{t_s} \frac{1}{(1 + S_w + S_f)} \tag{32}$$

Using a dimensional analysis procedure similar to that used in Ref.(8), Equations (28), (29) and (32) can be combined in the following form

$$\bar{\sigma} = \sigma_g^{2/5} \sigma_s^{1/5} \sigma_a^{2/5} \tag{33}$$

Performing the operations indicated in Equation (33) and using Equation (30) together with the definition of S_w given in Equations (26)

$$\bar{\sigma} = \frac{0.340\ \pi^{6/5} E}{(1-\nu^2)^{2/5}} \ \frac{[S_w^2 (S_w + 4) + 4 S_w S_f (S_w + 3)]^{1/5}}{(1 + S_w + S_f)^{4/5}} \ N/Ed^{2/5} \tag{34}$$

Optimum Solidity

By definition the solidity of a stiffened cylinder is the weight of the structure relative to the weight of a solid cylinder of the same diameter. In the optimization procedure to be employed, the solidity will be expressed in terms of fixed parameters and the stiffening system geometry. Then the solidity will be minimized with respect to the geometry. The result of the process will be an optimum geometry and an optimum solidity. From the derived optimum parameters, the optimum proportions of the longitudinal stiffening system can be obtained.

Using the assumption that the longitudinal and ring stiffening systems have the same weight:

$$\Sigma = \frac{4}{d}\, t_s\, (1 + 2\, S_w + 2\, S_f) \tag{35}$$

Since the applied stress and the effective stress are the same, the sheet thickness factor in Equation (35) can be eliminated by using Equations (32) and (34). The result is a solidity expression of the form

$$\Sigma = \frac{4\,(1-\nu^2)^{2/5}}{0.340\ \pi^{6/5}} \ \frac{(1 + 2\, S_w + 2\, S_f)}{[(1 + S_w + S_f)\,(S_w^3 + 4S_w^2 + 4S_w^2\, S_f + 12 S_w\, S_f)]^{1/5}} \ (N/Ed)^{3/5} \tag{36}$$

Implied in Equation (36) are minima of solidity with respect to the web weight, S_w, and the flange weight S_f, respectively. The absolute minimum solidity corresponds to that value where the minima with respect to the two independent weights coincide. For the web, minimum solidity occurs where $\partial\Sigma / \partial S_w = 0$ and for the flange $\partial\Sigma / \partial S_f = 0$. The results of these operations will be two equations in S_w and S_f which when solved simultaneously will yield the appropriate values of $(S_w)_o$ and $(S_f)_o$.

From the operation $\partial\Sigma / \partial S_w = 0$

$$2 S_w^4 + 4 (3S_f + 4) S_w^3 + 3 [24S_f - (S_f + 1)(2S_f - 3)] S_w^2$$
$$+ 8[12S_f(S_f + 1) + (S_f + 1)^2 (2S_f + 1) + 3S_f(S_f + 2)] S_w - 12S_f(1 + S_f)(1 + 2S_f) = 0 \tag{37}$$

and from $\partial\Sigma / \partial S_f = 0$

$$24(S_w + 3) S_f^2 + 8 [S_w(S_w + 4) + (S_w + 3)(6S_w + 3)] S_f$$
$$- [S_w(S_w + 4)(8S_w - 9) + 4(2S_w + 1)(S_w + 3)(S_w + 1)] = 0 \tag{38}$$

It is a reasonable approximation that the flange weight will be less than unity. An examination of Equation (37) indicates that for $0 > S_f > 1$ only one positive value of S_w exists. This value together with the lower positive value of the root of the quadratic Equation (38) results in the minimum solidity. A graphical solution of Equations (37) and (38) for these values results in:

$$(S_w)_o = 0.41 \qquad (S_f)_o = 0.104 \tag{39}$$

Substituting the optimum values in the solidity expression, Equation (36) and allowing $\nu = 0.3$:

$$\Sigma_o = 5.00 \, (N/Ed)^{3/5} \tag{40}$$

It is interesting to note that the solidity is quite insensitive to changes in the web or flange weights in the region of the optimum value. For example the changes in Σ_o are less than one percent for a variation of S_w from 0.35 to 0.45 and for a variation of S_f from 0.050 to 0.200.

Optimum Proportions

With the optimum flange and web weights known, it is now possible to determine other optimum parameters relative to the cylinder diameter. These are given below and also shown in Figures 6 and 7.

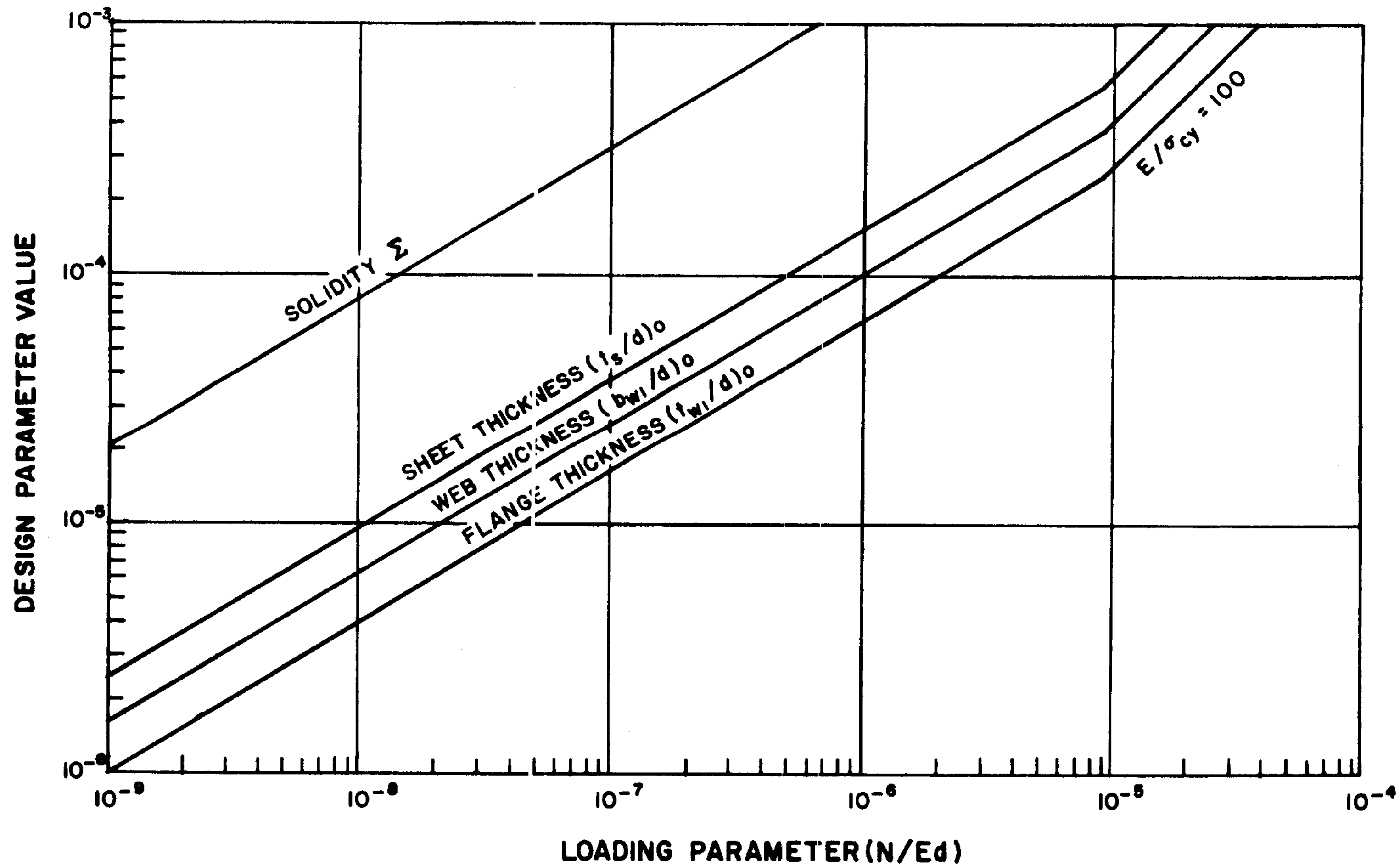


FIG. 6

OPTIMUM LONGITUDINAL STIFFENER THICKNESS AND OPTIMUM SOLIDITY

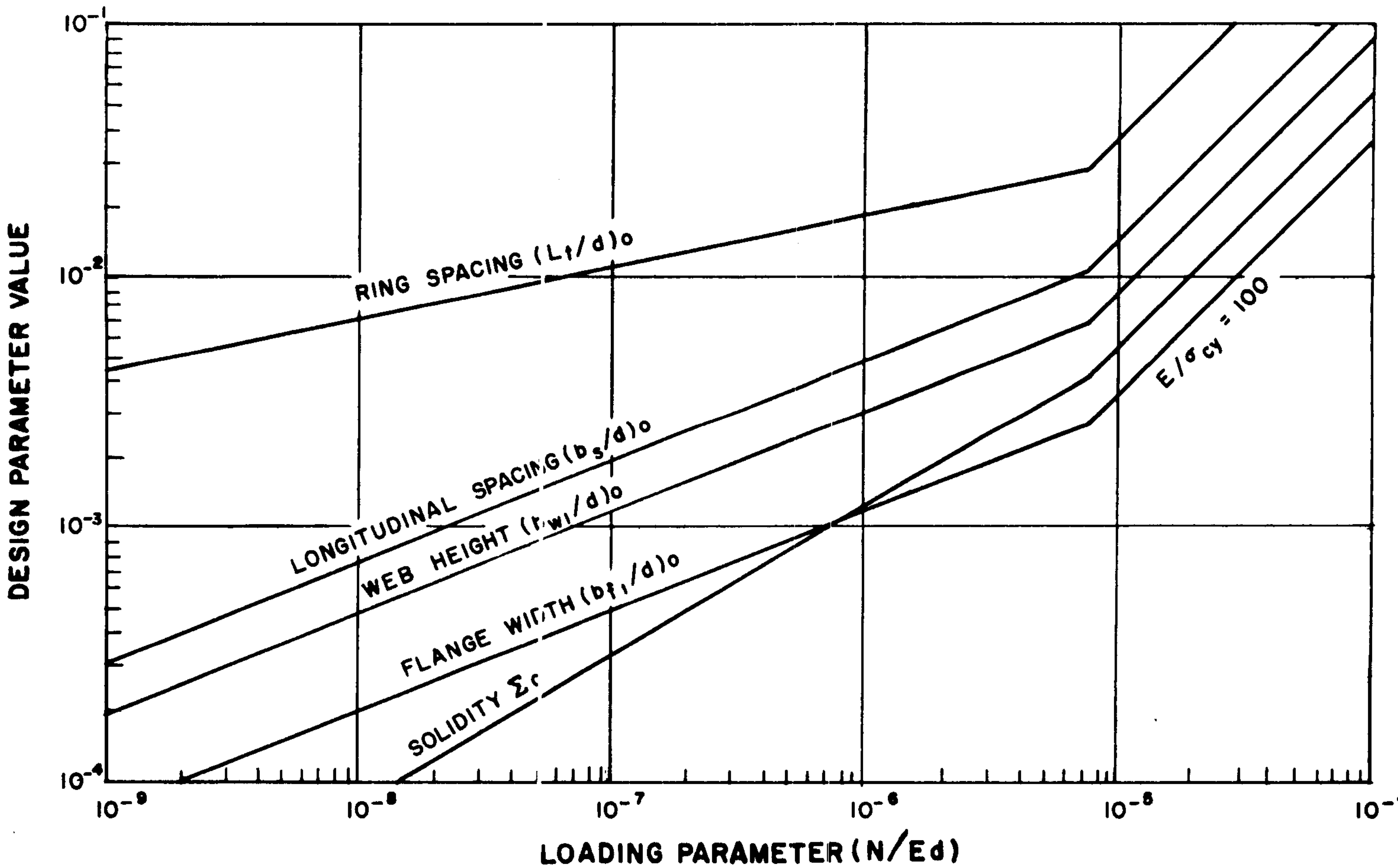


FIG. 7

OPTIMUM STIFFENER SPACING, LONGITUDINAL WEB HEIGHT AND FLANGE WIDTH, AND OPTIMUM SOLIDITY

Sheet thickness: $(t_s/d)_o = 0.617\ (N/Ed)^{3/5}$ (41)

For the longitudinal stiffening system:

Web height: $(b_{w1}/d)_o = 0.743\ (N/Ed)^{2/5}$ (42)

Web thickness: $(t_{w1}/d)_o = 0.401\ (N/Ed)^{2/5}$ (43)

Flange width: $(b_{f1}/d)_o = 0.301\ (N/Ed)^{2/5}$ (44)

Flange thickness: $(t_{f1}/d)_o = 0.248\ (N/Ed)^{3/5}$ (45)

Stiffener spacing: $(b_s/d)_o = 1.162\ (N/Ed)^{2/5}$ (46)

The lines in Figures 6 and 7 with the steeper slope represent the yield strength cut off values of the parameters for a value of $E/\sigma_{cy} = 100$.

The ring spacing is obtained by assuming that the structure between rings buckles as a wide column with a stress equal to the optimum cylinder stress. This value is called the optimum ring spacing although the stress value may not be the optimum stress for a stiffened wide column when used as such. In this case, however, the cylinder stress is allowed to govern the design assuming $\sigma_{co} = \sigma_o$ then

$$(L_t)_o = \pi(D_1/t_1\ \sigma_o)^{\frac{1}{2}} \tag{47}$$

Substituting the appropriate optimum parameters in Equation (47)

Ring spacing: $(L_t/d)_o = 0.258\ (N/Ed)^{1/5}$ (48)

The ring stiffener geometry cannot be determined from the optimum analysis. However, the axisymmetric buckling criterion can be invoked in order to obtain the ring geometry. For rectangular cross section rings equal in weight to the total weight of the longitudinal stiffeners, from Equation (1)

$$\alpha = \frac{b_{w2}^2}{b_{w1}^2} \quad \frac{S_2\,(4 + S_2)}{S_{w1}(4 + S_{w1}) + 4S_{f1}(3 + S_{w1})} \tag{49}$$

Since $S_2 = S_{w1} + S_{f1}$, the optimum values can be substituted in Equation (49) and

$$\alpha = 0.727\ (b_{w2}/b_{w1})^2 \tag{50}$$

The pertinent ring dimensions using Equation (50) and the definition of S_2:

$$\text{Ring height:} \qquad b_{w2}/d = 0.633\alpha^{\frac{1}{2}} \, (N/Ed)^{2/5} \tag{51}$$

$$\text{Ring thickness:} \quad t_{w2}/d = (0.1305/\alpha^{\frac{1}{2}})(N/Ed)^{2/5} \tag{52}$$

where the condition $\alpha > 1$ is necessary to assure axisymmetric buckling.

Design values of the ring height and thickness are given in Figure 8 for $\alpha = 1$ and $\alpha = 10$. The value $\alpha = 1$ would not be used for design purposes and has been included only as a reference value.

For tee shape cross section rings equal in total weight to that of the longitudinals, the mode index expression, Equation (1) becomes

$$\alpha = \frac{b_{w2}^2}{b_{w1}^2} \left[\frac{S_{w2}(4 + S_{w2}) + 4S_{f2}(3 + S_{w2})}{S_{w1}(4 + S_{w1}) + 4S_{f1}(3 + S_{w1})} \right] \tag{53}$$

It is apparent from the form of Equation (53) that the ring stiffening system cannot be completely fixed for given values of α. A relation between the weights of the flange and the web cannot be obtained from buckling considerations since the ring does not carry any of the axial load. A convenient assumption to make is that the flange weight is a fraction of the web weight:

$$S_{w2} = nS_{f2} \tag{54}$$

This assumption together with the original weight assumption given in Equation (27) when used in Equation (53) with the previously determined optimum weights results in an expression for the ring web height in the form

$$b_{w2} = \alpha^{\frac{1}{2}} \frac{(1 + n)}{(2.66n^2 + 4.85n + 1.93)^{\frac{1}{2}}} \, b_{w1} \tag{55}$$

For given values of α and n, the ring geometry becomes fixed. For example if $n = 4$, the ring stiffeners have the same relative geometry as the longitudinals and Equation (55) becomes

$$b_{w2} = \alpha^{\frac{1}{2}} \, b_{w1} \tag{56}$$

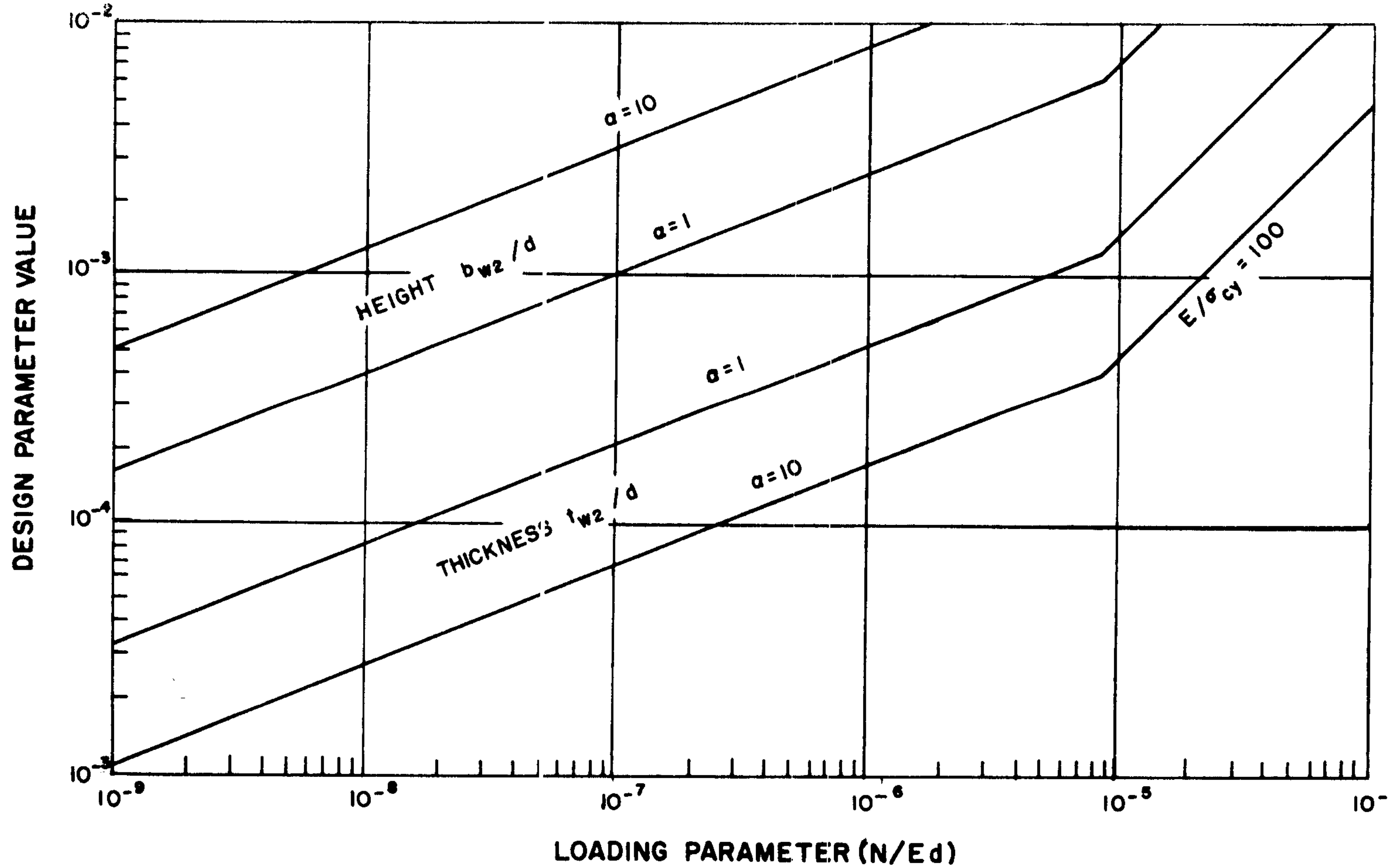


FIG. 8

DESIGN GEOMETRY FOR RECTANGULAR RINGS

The ring dimensions then become

$$\text{Ring web height:} \qquad b_{w2}/d = 0.743\ \alpha^{\frac{1}{2}}\ (N/Ed)^{2/5} \tag{57}$$

$$\text{Ring web thickness:} \qquad t_{w2}/d = (0.090/\alpha^{\frac{1}{2}})(N/Ed)^{2/5} \tag{58}$$

These design values are shown graphically in Figure 9.

The flange dimensions cannot be obtained explicitly. Rather the restriction on the flange is simply that it shall have the same relative weight as the comparable longitudinal element in the example chosen. This requirement fixes the area of the ring flange as:

$$\text{Ring flange area:} \qquad b_{f2}\, t_{f2}/d^2 = 0.0166\,(N/Ed)^{4/5} \tag{59}$$

The flange area variation with loading parameter is shown graphically in Figure 10.

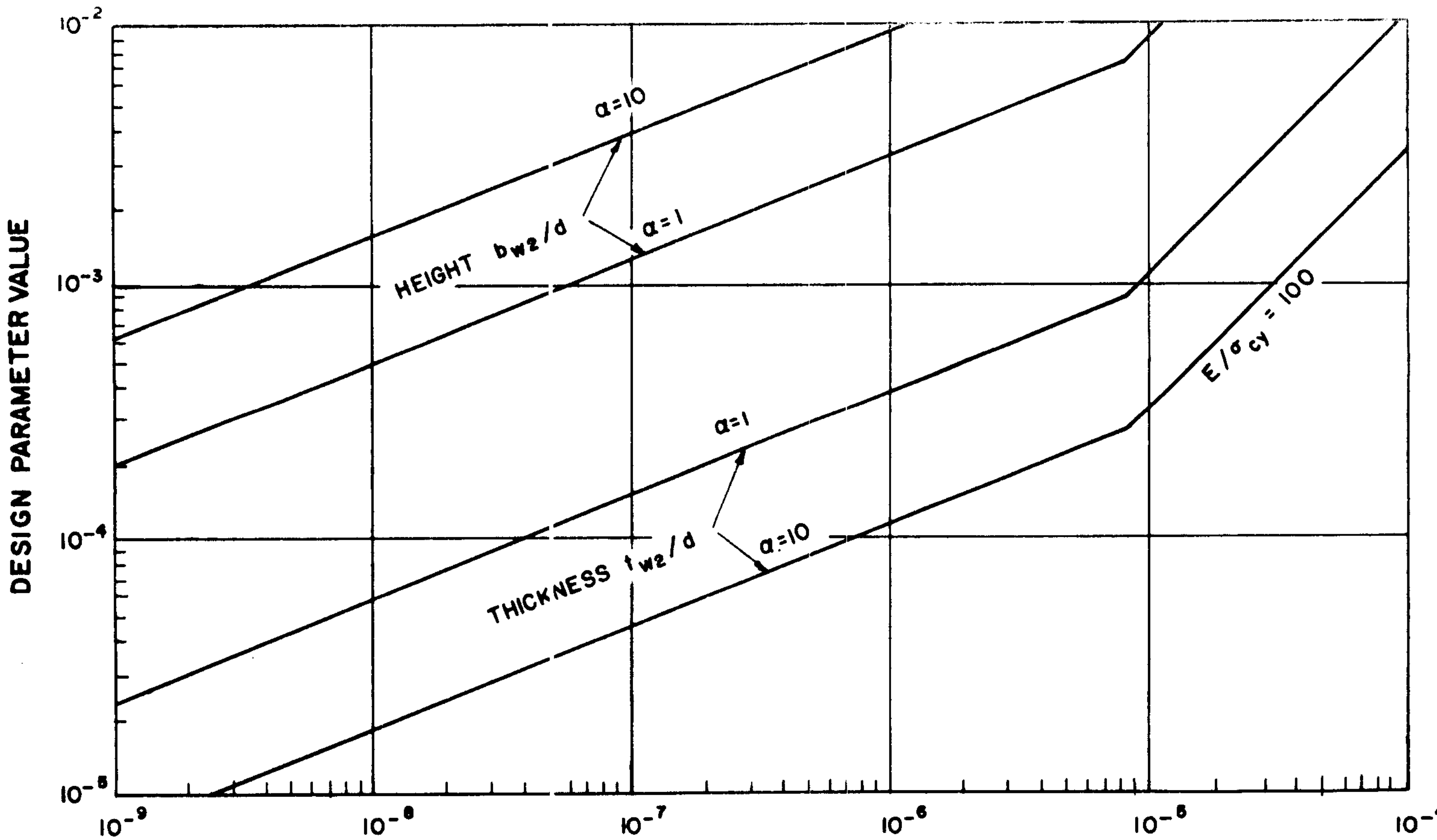


FIG. 9

WEB DESIGN GEOMETRY FOR TEE RING WHERE $S_{w2} / S_{f2} = 4$

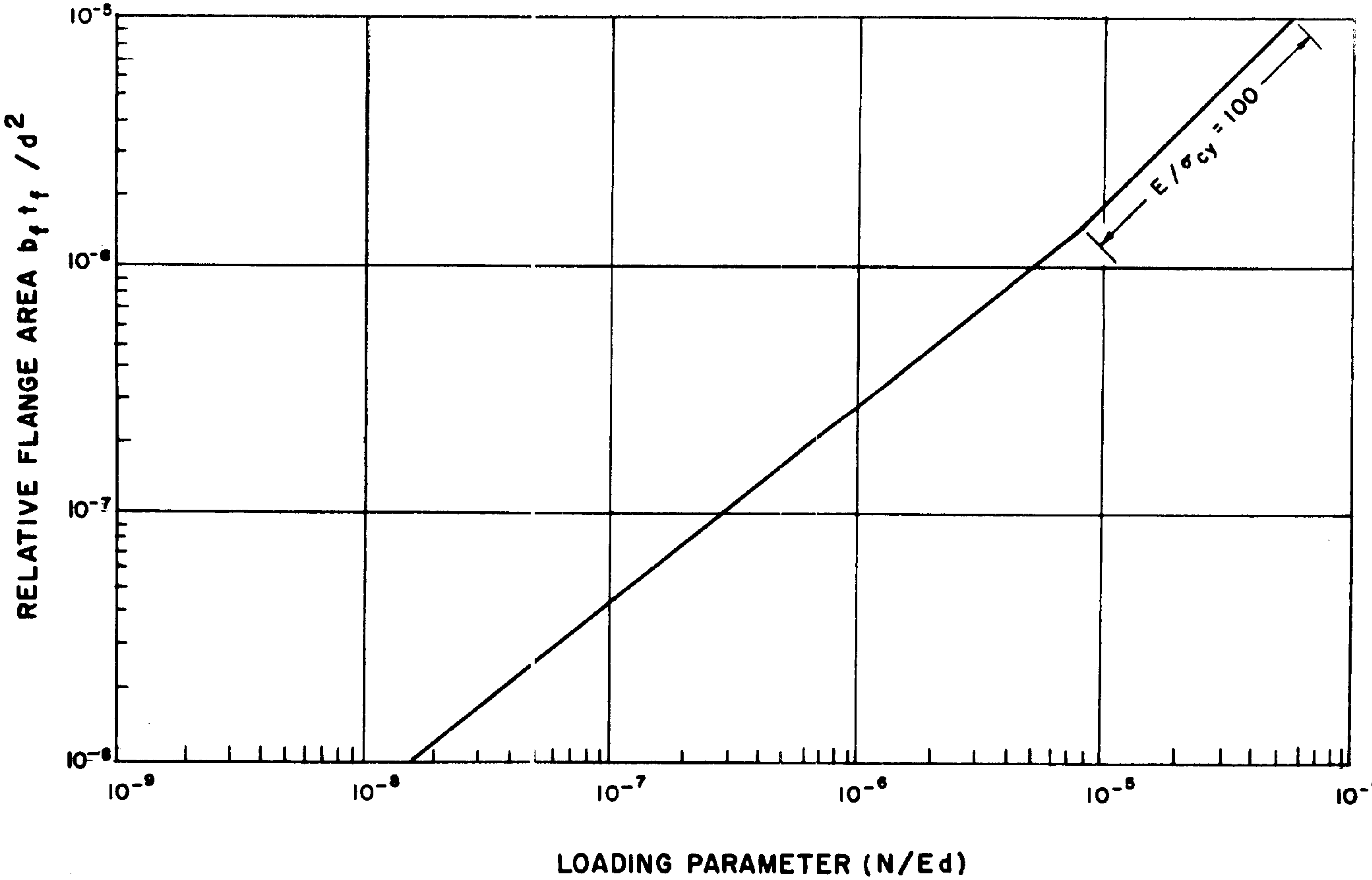


FIG. 10

FLANGE DESIGN AREA FOR TEE RINGS WHERE $S_{W2}/S_{f2} = 4$

4. COMPARATIVE EFFICIENCIES OF STIFFENED CYLINDERS

Introduction

Methods have been developed to determine the optimum proportions of many different types of stiffened structures under compressive loading such that a minimum weight design results (see for example Refs. 2 and 3). At present, however, no principles have been elucidated for arriving directly at an optimum stiffening system configuration for a particular application. Rather it is the ingenuity of the designer which is the source of the configuration and after a particular configuration has been optimized, its weight is compared with other optimized systems to determine the most efficient design.

It is with this latter process for stiffened, moderate length cylinders in axial compression that this part is concerned. Minimum weight designs have been developed for several grid stiffened systems in Parts 2 and 3. These will be compared with one another as well as with efficient designs for unstiffened, ring stiffened, sandwich and pressure stabilized cylinders for an overall assessment of stiffened cylinder efficiency.

Optimum Designs

The function of the rings in a properly designed ring stiffened, moderate length cylinder is to constrain the structure to buckle in the axisymmetric mode. The stiffeners themselves theoretically carry none of the compressive load of the cylinder and do not buckle. This is in contrast to the longitudinally stiffened system where the longitudinal stiffeners share the load carrying function with the skin. As a consequence of this difference in stiffener behavior it is found that no optimum design for the rings can be obtained, only an efficient design which will meet the axisymmetric buckling criterion.

Efficiencies of various cylinders in axial compression are compared by comparing the solidities as is done in Figure 11 and in Table 1. These values are based upon elastic buckling and hence are valid in the elastic region of the stress strain curve. For a typical high strength material the yield strength limit for $E/\sigma_{cy} = 100$ is indicated in the figure by the line at 45°.

For the unstiffened cylinder, the instability relation which is supported by test data (9) is one in which the buckling coefficient is a function of the

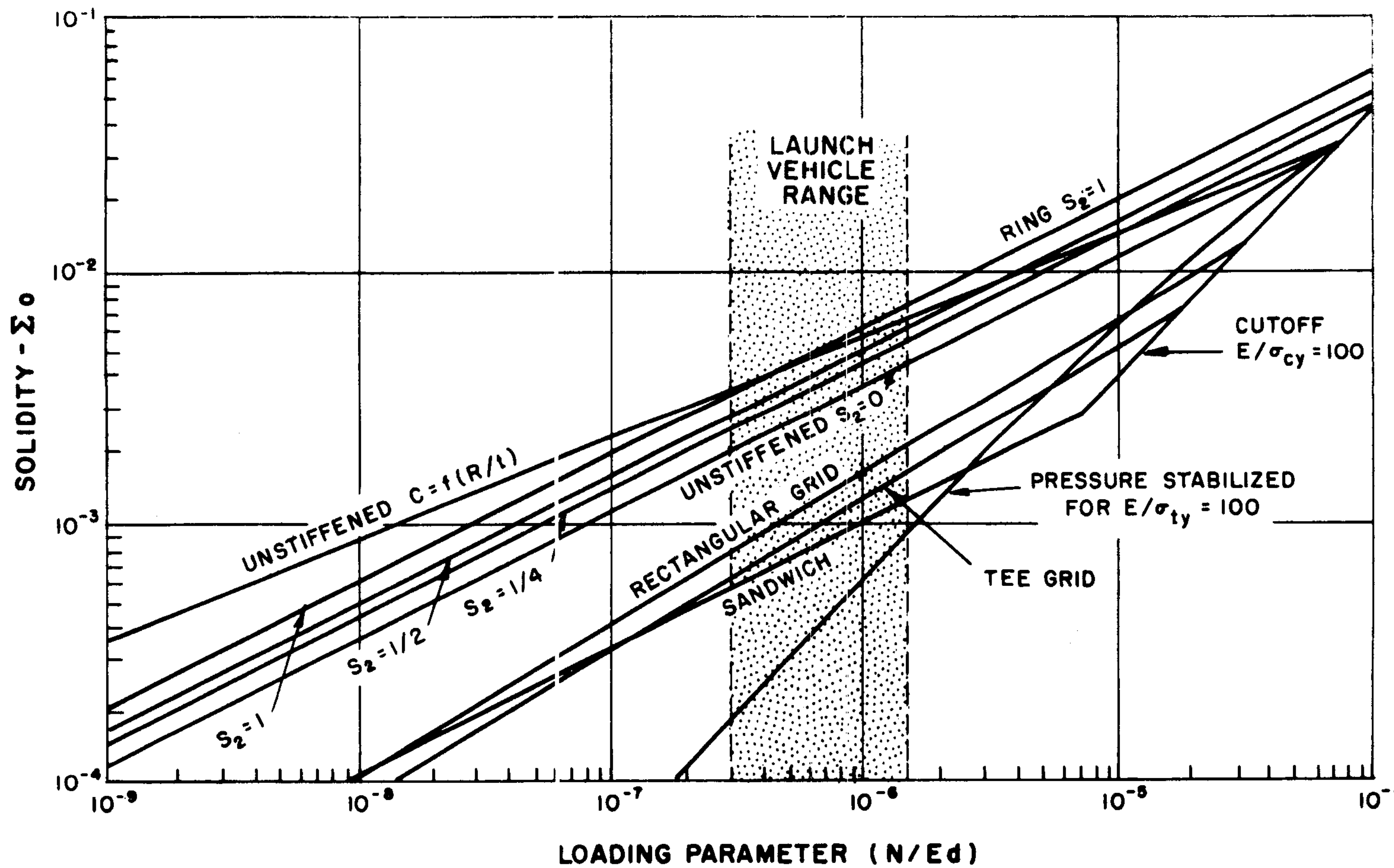


FIG. 11

COMPARATIVE EFFICIENCIES OF VARIOUS STIFFENED CYLINDERS radius to thickness ratio. The resulting solidity is the upper line in Figure 11.

Considering ring stiffened cylinders, one can use a relation of the form given in Equation (22) to determine the critical stress as a function of stiffener weight. Solidities based upon ring weights $S_2 = 0$, 1/4, 1/2 and 1 respectively are shown in Figure 11. Using $S_2 = 0$ in Equation (22) results in an instability relation for an unstiffened cylinder and the resulting solidity is the absolute theoretical lower limit for ring stiffened cylinders. This reference case is unrealistic in practice, however, since it assumes that the unstiffened cylinder will buckle in the axisymmetric mode.

Comparing the experimentally verified unstiffened case with the ring stiffened cylinders indicates no particular advantage in ring stiffening for high values of the loading parameter in Figure 11. It is only under very low loadings that the ring stiffened cylinders are advantageous.

A more efficient version of the circumferential type of stiffening system is one which contains longitudinals between the rings, the grid stiffened types considered in Parts 2 and 3. The solidities for rectangular cross section longitudinals (Part 2) and tee cross section longitudinals (Part 3) computed on the basis of equal weight longitudinals and rings ($S_1 = S_2$) are shown in Figure 11. These are considerably more efficient than any of the ring stiffened configurations.

Shown also in the figure are data for the particular sandwich cylinder (10) where the density of the faces is 50 times the density of the core. This configuration is only superior to the grid stiffened for very high loadings which result in stresses close to the yield strength cut off.

The results of an analysis of pressure stabilized cylinders (10) for materials where $E/\sigma_{ty} = 100$ as given in Figure 11 show that the pressure stabilized case shows significant improvement in efficiency over the stiffened and unstiffened cylinders for the lower values of the loading parameter.

Launch Vehicle Designs

The comparative efficiencies of various types of stiffening systems are presented for a broad range of N/Ed values in Figure 11. It remains now to relate the N/Ed values pertinent to launch vehicle design, to the data presented in Figure 11 in order to draw some important generalized conclusions.

A compilation of available data on past, present and projected launch vehicles given in Table 2 indicates that although the compressive loadings vary by a factor of over 300 and the diameters by a factor of 13, the loading index N/Ed varies only by a factor of 5! In fact, as shown in Figure 11, the variation in the latter is sufficiently small that a value of N/Ed of 10^{-6} can be taken as representative of current launch vehicle designs.

An exploratory study of current and projected space vehicle designs has led to the tentative conclusion that launch and/or pressurization loads appear to govern their primary structural design. Furthermore, a review of a small amount of data on payload structures indicates that the N/Ed launch vehicle range may also be representative of such structures. The preliminary data remain to be further substantiated by detailed analysis of other space vehicles.

Generalized Conclusions

The identification of the launch vehicle design range in Figure 11 permits the following generalized conclusions to be identified:

1. The N/Ed range of current and projected launch vehicles is such that elastic buckling considerations govern if reasonable compressive yield strength materials are utilized. Because elastic buckling governs the lower density alloys becomes desirable (except for the pressure stabilized case).

2. On the basis of compressive loading as the design criterion there is no advantages in using high strength sheet materials for the primary launch vehicle structure (except for the pressure stabilized case) since the N/Ed range is relatively low. In fact, aluminum alloys with a compressive yield strength of 50 ksi should be quite adequate.

3. In the launch vehicle N/Ed range shown in Figure 11, optimum grid stiffened cylinders are roughly one-quarter of the weight of unstiffened cylinders. Moreover, they are directly competitive with optimum sandwich cylinders.

4. Pressure stabilized cylinders that utilize high strength sheet materials ($E/\sigma_{ty} = 100$) are distincly superior to other forms of construction at the lower end of the launch vehicle N/Ed range. From a materials viewpoint, the efficiency of pressure stabilized structures depends upon the tensile strength/density ratio.

TABLE 1. COMPARATIVE WEIGHTS FOR ELASTIC BUCKLING OF STIFFENED CYLINDERS UNDER AXIAL COMPRESSION

Stiffening System	Solidity - Σ
Isotropic $C = f(R/t)$	$1.40\ (N/Ed)^{2/5}$
Isotropic $S_2 = o$	$3.63\ (N/Ed)^{\frac{1}{2}}$
Ring $S_2 = \frac{1}{4}$	$4.30\ (N/Ed)^{\frac{1}{2}}$
$S_2 = \frac{1}{2}$	$4.93\ (N/Ed)^{\frac{1}{2}}$
$S_2 = 1$	$6.12\ (N/Ed)^{\frac{1}{2}}$
Grid - Rectangular	$6.45\ (N/Ed)^{3/5}$
Grid - Tee	$5.00\ (N/Ed)^{3/5}$
Sandwich	$1.02\ (N/Ed)^{\frac{1}{2}}$

TABLE 2. COMPRESSIVE LOADING INDICES FOR LAUNCH VEHICLES

Vehicle	Thrust-lb	Diameter-in	N/Ed
Redstone	0.078×10^6	70	5.10×10^{-7}
Thor	0.170	96	5.87
Atlas	0.389	120	2.86
Minuteman	0.170	71	3.58
Titan I	0.300	120	7.95
Titan II	0.430	120	11.4
Saturn V	7.50	400	14.9
Nova	25	960	8.6

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